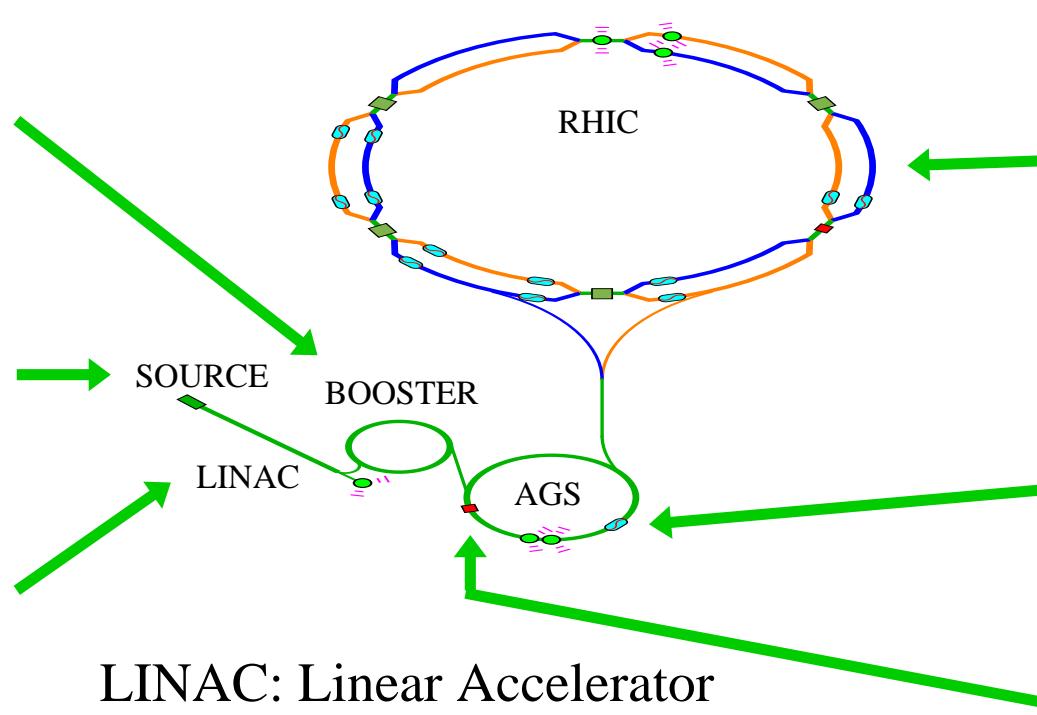


eRHIC: Polarized Ions

Waldo MacKay



Accelerator Complex (Pol. Protons)



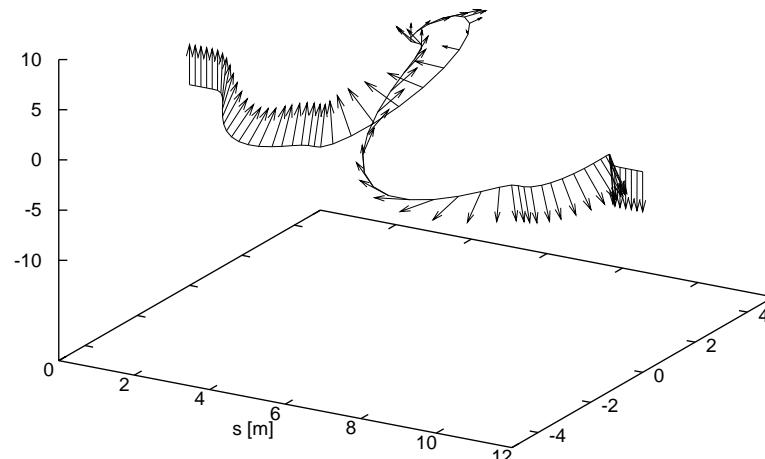
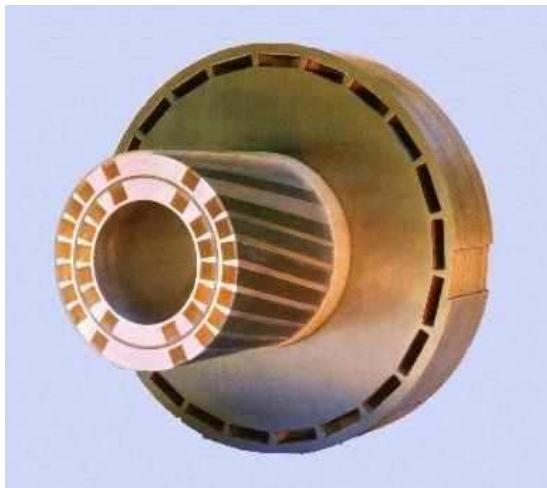
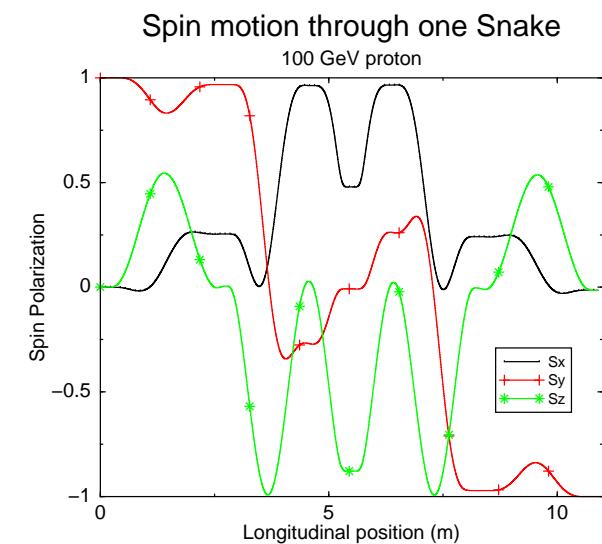
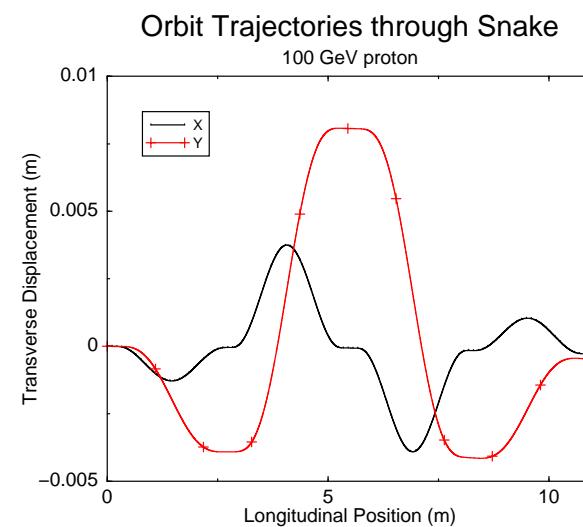
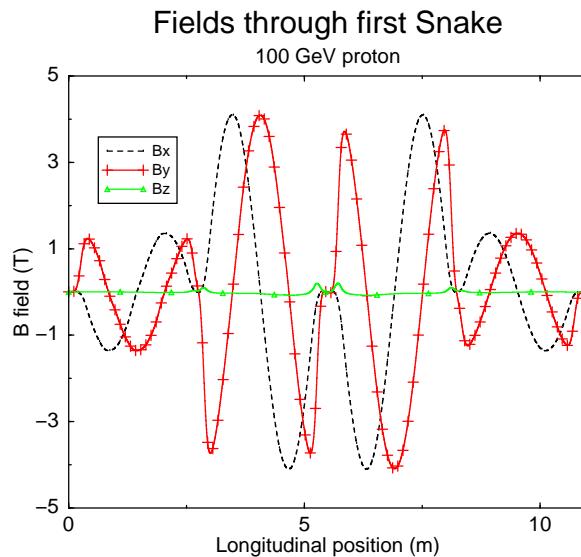
LINAC: Linear Accelerator

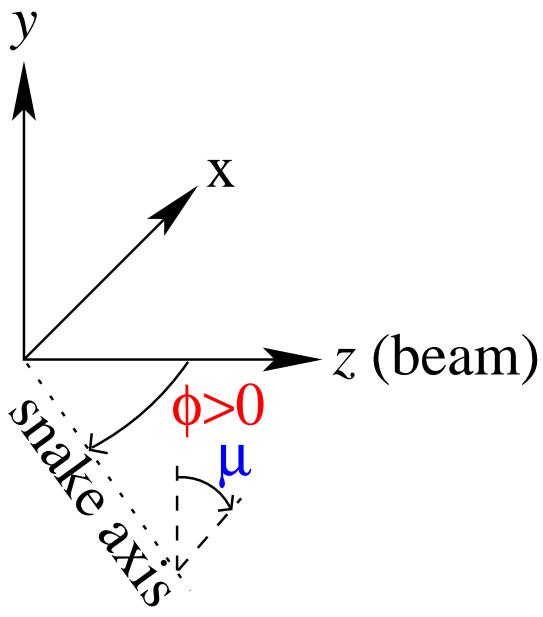
AGS: Alternating Gradient Synchrotron

RHIC: Relativistic Heavy Ion Collider

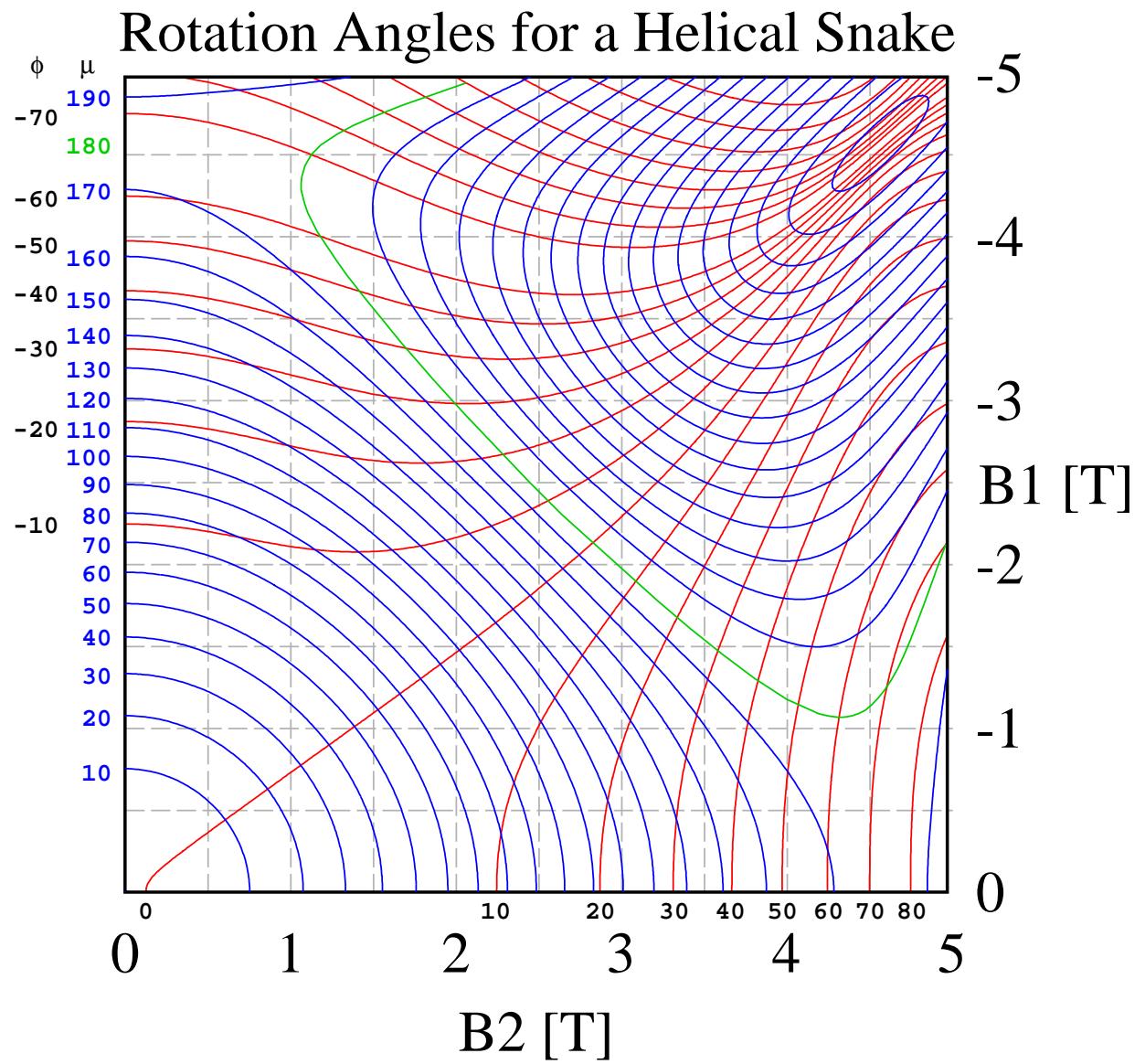


_trajectory and Spin through Snakes

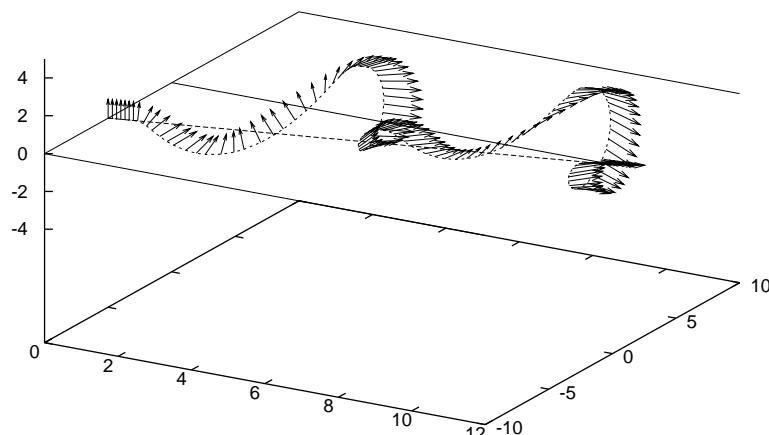
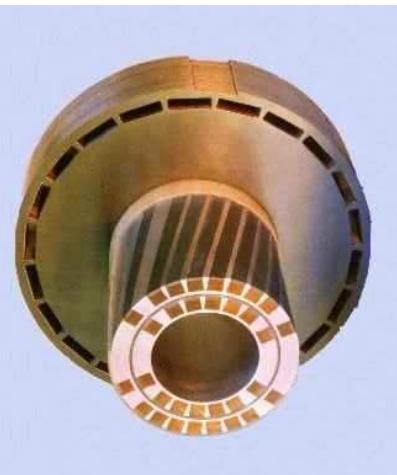
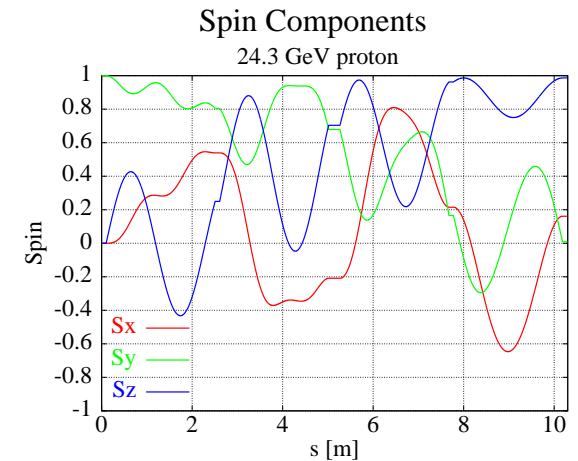
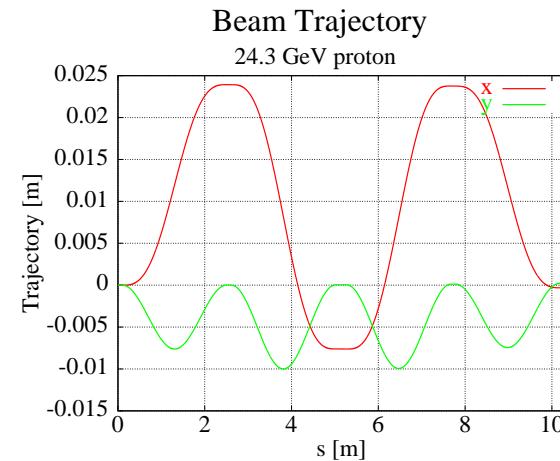
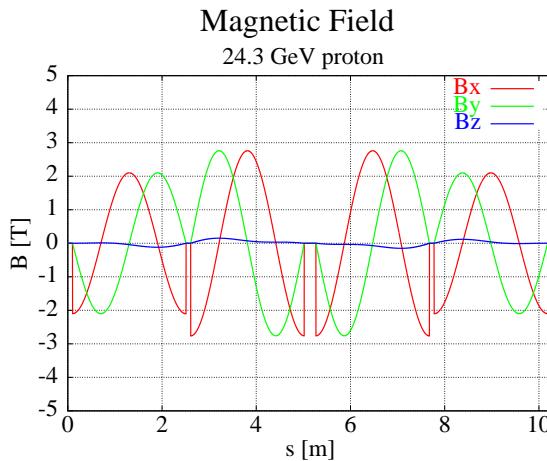


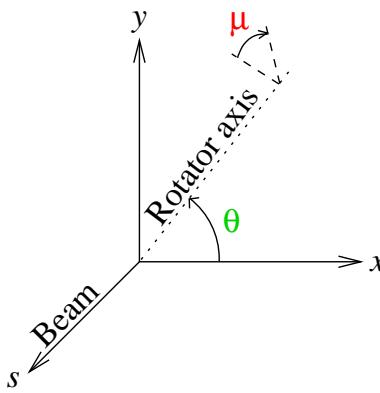


The snake's axis of rotation is in the horizontal plane at an angle ϕ from the z -axis, and μ is the left-handed rotation angle.



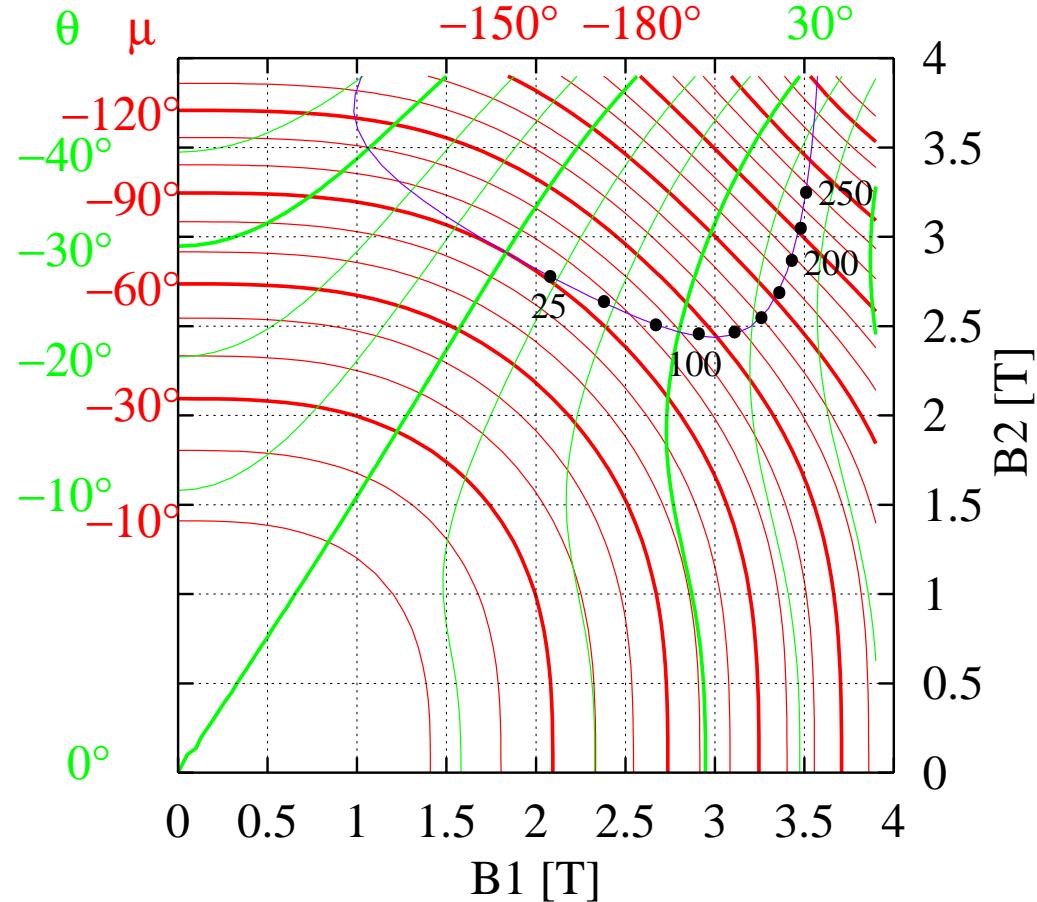
⚡ Helical Spin Rotators ⚡





The rotation axis of the spin rotator is in the x - y plane at an angle θ from the vertical. The spin is rotated by the angle μ around the rotation axis.

Rotation Angles for a Helical Spin Rotator



Note: Purple contour for rotation into horizontal plane.
Black dots show settings for RHIC energies in increments of 25 GeV from 25 to 250 GeV.

§ Rotator Axes and Precession §

To precess the spin from vertical into the horizontal plane:

$$\sin \beta = \sin \mu \cos \theta$$

$$\cos \mu = -\tan^2 \theta$$

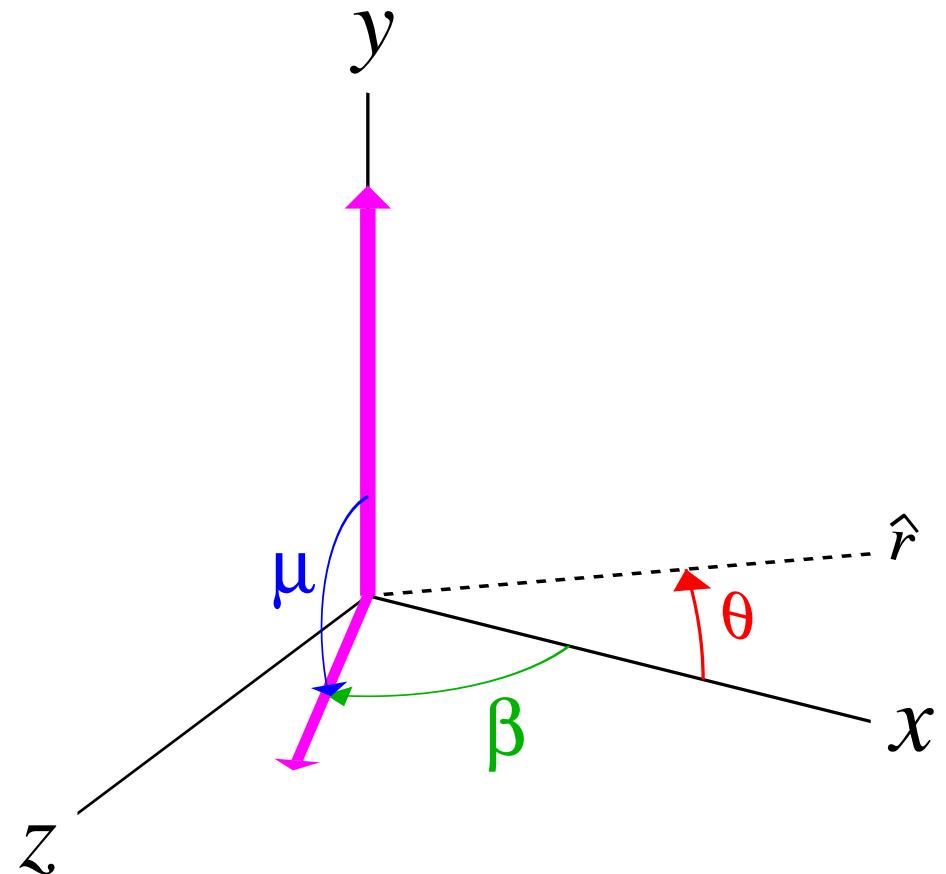
$$\mu \in [90^\circ, 270^\circ]$$

$$\theta \in [-45^\circ, 45^\circ] \cup [135^\circ, 225^\circ]$$

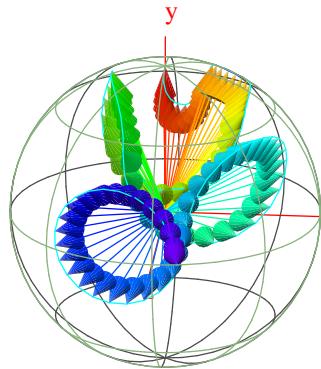
For longitudinal polarization want:

$$\beta = G\gamma \times \theta_{D0DX}$$

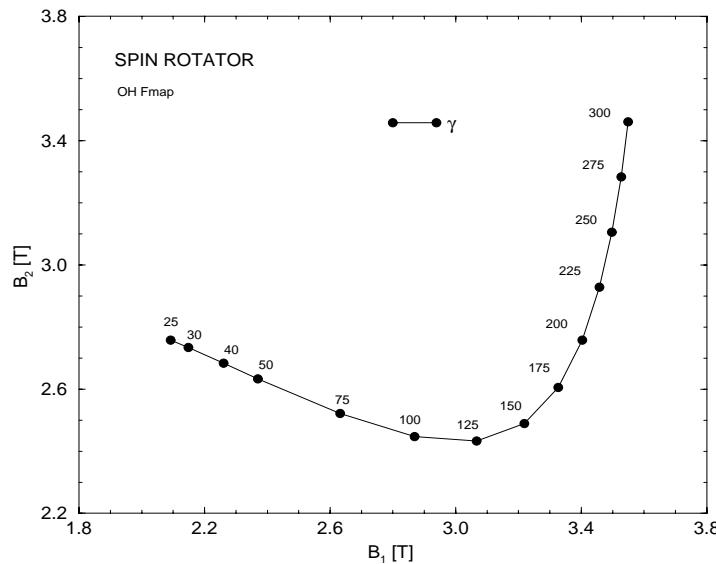
$$\theta_{D0DX} = 3.675 \text{ mr}$$



Compensation for D0-DX Bends

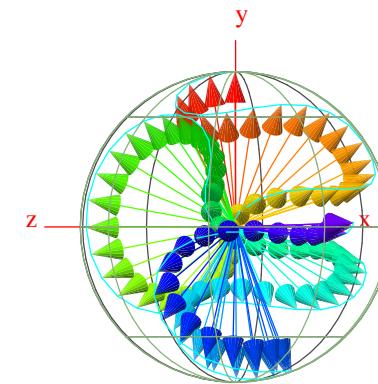


Rotator's spin vector at injection energy



$E = 25$ GeV

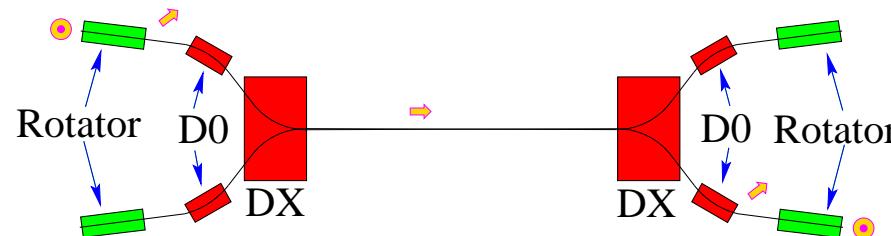
D0DX: 10° precession



Rotator's spin vector at 250 GeV

$E = 250$ GeV

D0DX: 100° precession



Formulae for a Single Rotator Helix

Parameters for a single RHIC rotator helix

Pitch: $k = \frac{2\pi}{\lambda}$, $\lambda = 2.41$ m [$+(-)$ for right(left)-handed]

$$\kappa = \frac{q}{p}(1 + G\gamma)B$$

Rotation axis: $\hat{r} = \frac{k\hat{z} + \kappa\hat{x}}{\sqrt{\kappa^2 + k^2}}$

Precession angle: $\alpha = 2\pi \left(\sqrt{1 + \left(\frac{\kappa}{k}\right)^2} - 1 \right)$

Transverse offset: $\Delta x = \frac{q}{p} \frac{B\ell}{k} = \frac{q}{p} \frac{\lambda^2}{2\pi} B$

Scaling Snakes and Rotators to He³

Scaling of the field at maximum energy:

The maximum rigidity of the beams must be the same: $r_{\max} = \frac{p}{q} = 834$ Tm

$$(\beta\gamma)_{\text{He}^3} \simeq \frac{Z}{A} (\beta\gamma)_p$$

Want the same precession, so κ must be the same.

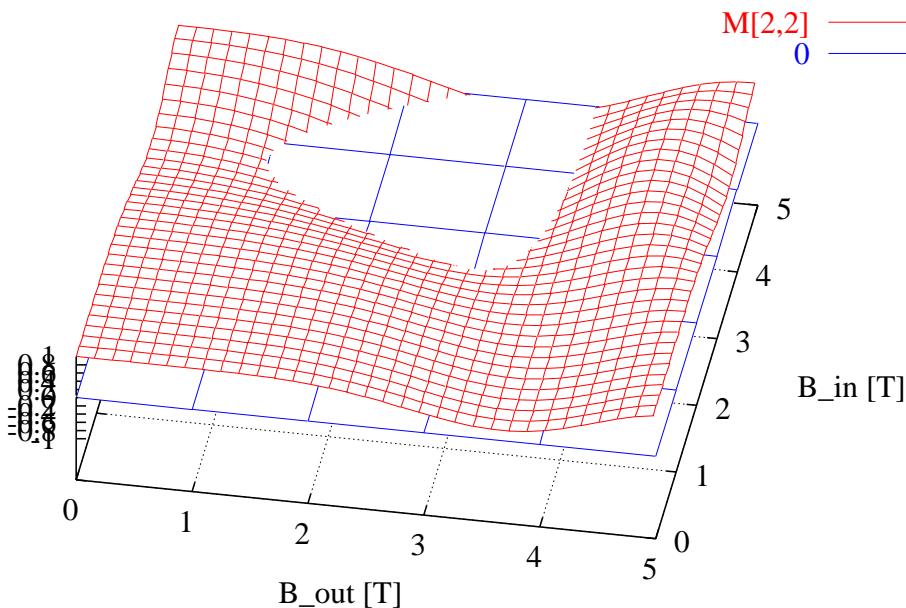
$$\begin{aligned} B_{\text{He}^3} &\simeq \frac{1 + G_p \gamma_p}{1 + G_{\text{He}^3} \gamma_{\text{He}^3}} B_p \\ &\simeq \frac{AG_p}{ZG_{\text{He}^3}} B_p \simeq -0.643 B_p \end{aligned}$$

Snake excursion at injection $r_{\text{inj}} = 81.1$ Tm (for protons):

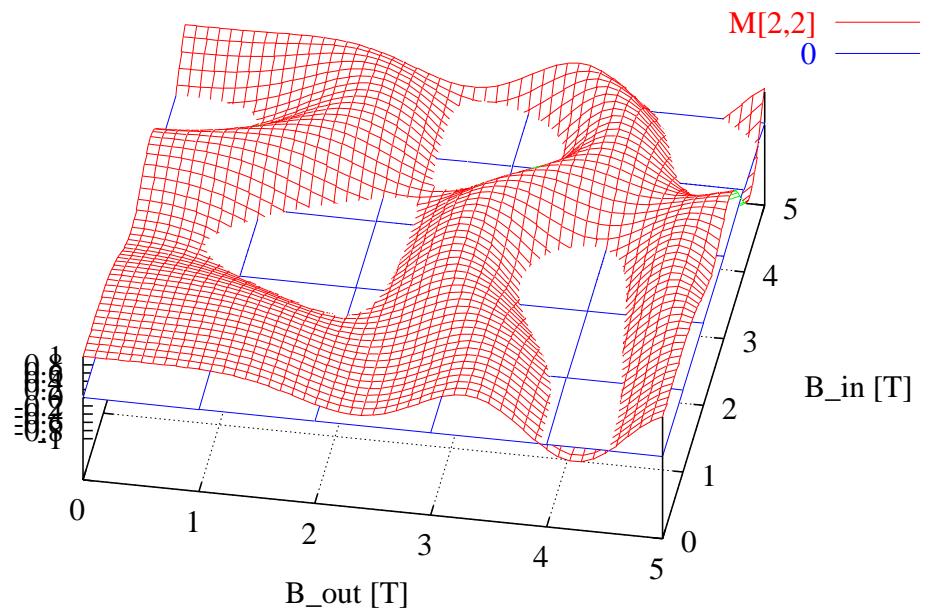
$$\Delta y = \begin{cases} 3.2 \text{ cm,} & \text{for protons} \\ -2.1 \text{ cm,} & \text{for He}^3 \end{cases}$$

Comparison of Rotators for He³ and p

Spin rotator contours for protons



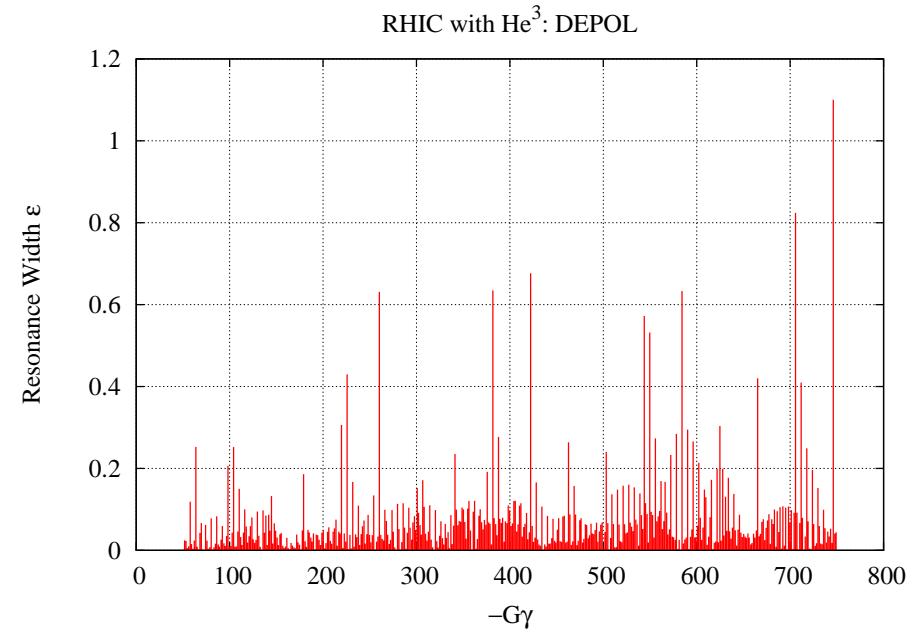
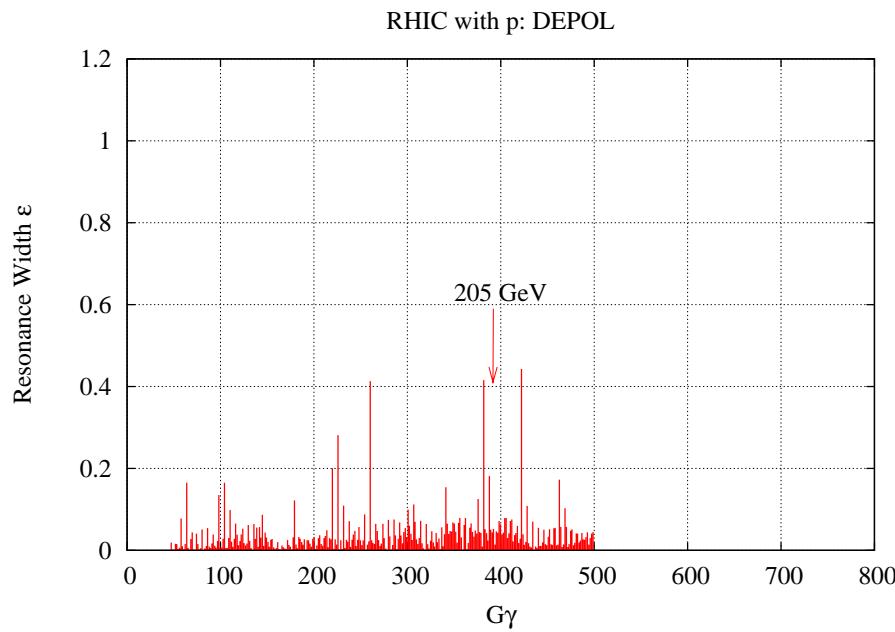
Spin rotator contour for He3



RHIC Spin Params for Diff. Species

	p	$^2\text{H}^+$	$^3\text{He}^{+2}$	e^-
$m [\text{GeV}/c^2]$	0.9382720	1.8756127	2.8083912	0.0005109989
$G = (g - 2)/2$	1.79284734	-0.1426177	-4.184	0.001159652
$mc^2/G [\text{MeV}]$	523.3418	13156.49	671.2216	440.6485
$(p/q)_{\text{inj}} [\text{Tm}]$	81.113	81.113	81.027	
$U_{\text{inj}} [\text{GeV}]$	24.335	24.364	48.664	
$U_{\text{inj}}/n [\text{GeV}]$	24.335	12.182	16.221	
γ_{inj}	25.9362	13.0034	17.3280	
$G\gamma_{\text{inj}}$	46.500	-1.854	-72.500	
$(p/q)_{\text{store}} [\text{Tm}]$	833.904	833.904	833.904	33.356
$U_{\text{store}} [\text{GeV}]$	250.000	250.005	500.004	10
$U_{\text{store}}/n [\text{GeV}]$	250.000	125.003	166.668	10
γ_{store}	266.4473	133.2926	178.0394	19569.54
$G\gamma_{\text{store}}$	477.699	19.062	744.917	22.6938

↳ Intrinsic Resonances in RHIC ↳



$$|G\gamma|_{\max} = \begin{cases} 477, & p \\ 743, & \text{He}^3 \end{cases}$$

Calculations courtesy of M. Bai

Injection–Extraction for He3

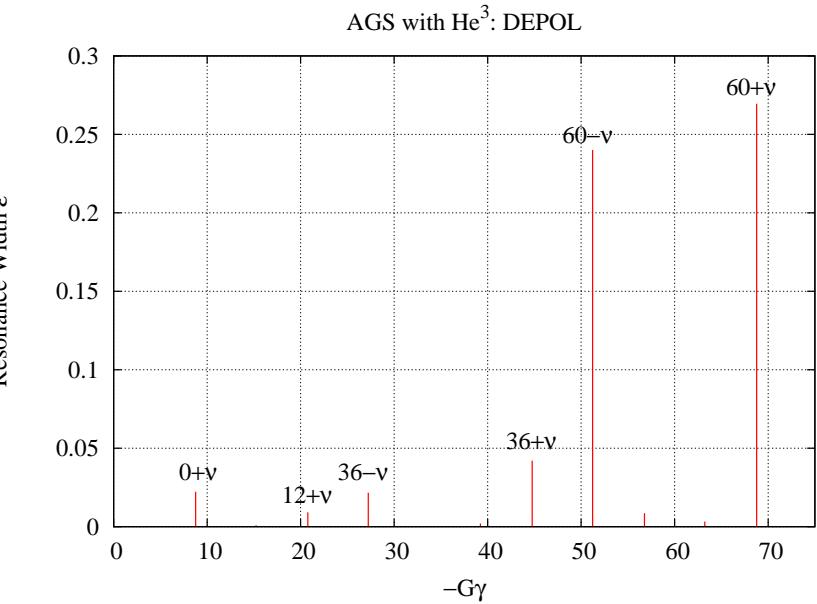
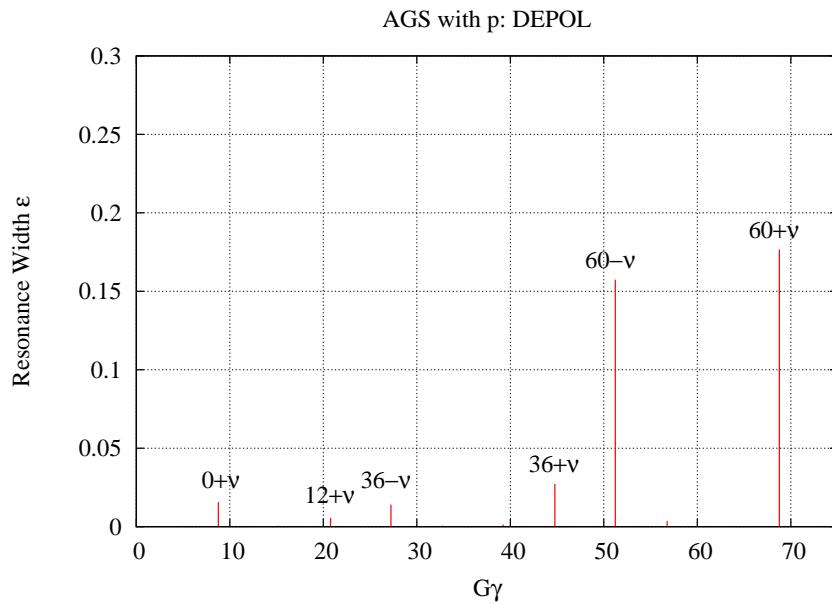
Values corresponding to proton rigidities

	$U/n[\text{GeV}]$	γ	$G\gamma$	$p/q[\text{Tm}]$
Booster inj ^a	0.9381	1.0021	-4.1929	0.3063
Booster inj ^b	0.9421	1.0064	-4.2108	0.5312
AGS inj	1.7176	1.8348	-7.6766	7.2052
RHIC inj	16.2385	17.3464	-72.5775	81.1138
RHIC max	166.668	178.039	-744.917	833.904

^a kinetic energy: 2 MeV/nucleon from EBIS

^b kinetic energy: 6 MeV/nucleon from EBIS with extra cavities

Intrinsic Resonances in AGS



$$|G\gamma|_{\text{ext}} = \begin{cases} 46.5, & \text{p} \\ 72.5, & \text{He}^3 \end{cases}$$

$G\gamma = 50.5$ is just below the $60 - \nu$ resonance.
It corresponds to $p/q \simeq 56.4$ Tm.

He3 Spin Resonances in Booster

- Intrinsic resonances with present tunes:
 $0 + \nu$: $|G\gamma| = 4.54$ with $\varepsilon = 0.011$
 $12 - \nu$: $|G\gamma| = 7.46$ with $\varepsilon = 0.009$
- Can extract just before $12 - \nu$.
- Injection will be at $|G\gamma| = 4.187$ which is before $0 + \nu$
 - $|G\gamma| = 4.54$ corresponds to a kinetic energy of 239 MeV ($\gamma = 1.0851$).
⇒ Move Qv below 4.187 (2 MeV/n) or 4.21 (6 MeV/n).
 - Would allow for slightly higher extraction energy from Booster.
- There are also the imperfection resonances at 5, 6, and 7 to contend with.

↳ Summary ↳

- ~ Spin precession and orbit excursions in snakes and rotators should work for protons in eRHIC.
 - Snakes the same for protons.
 - If no “D0DX” bends for eRHIC IR, then fields in rotators are essentially constant for all energies (like the snakes).
 - Polarized protons now at 205 GeV!
- ~ He³ requires less field in snakes and rotators.
 - Injection orbit excursions reduced.
 - Source: EBIS with OPPIS and ion linac.
 - 2×10^{11} at source with 50–70% polarization.
 - $|G\gamma|_{\max}$ is higher for He³.
 - More and Stronger resonances in all rings:
 - Injection rigidity may be lower for RHIC ($< 60 - \nu$ in AGS).
 - AGS injection and extraction spin-matching more severe.